Journal of Mathematical Behavior 80 (2025) 101281

Contents lists available at ScienceDirect

Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

Enhancing mathematical instruction for emergent bilinguals through collaborative situated professional development

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ARTICLE INFO

Keywords:
Collaborative situated professional development
Emergent bilinguals
Mathematics instruction
Teaching practices
Mathematics teachers

ABSTRACT

This study investigates the shifts in high school mathematics teachers' instructional practices when working with emergent bilinguals (EBs) after participating in a collaborative situated professional development (CSPD) program alongside researchers. Over the course of one academic year, the teachers engaged in co-planning, co-teaching, and co-reflecting sessions with the research team while integrating research-based strategies to support EBs in learning mathematics. The CSPD also emphasized implementing cognitively demanding and contextually relevant mathematical tasks. The results reveal that the teachers' instructional practices significantly improved after participation in the CSPD. These improvements include a stronger ability to maintain the cognitive demand of tasks throughout lessons, an increased focus on creating opportunities for students to articulate and explain their mathematical reasoning, and more effective incorporation of meaningful visual aids that enhance comprehension for EBs. These shifts highlight the potential of CSPD to positively impact mathematics instruction for linguistically diverse classrooms.

1. Introduction

The increasing linguistic and cultural diversity in classrooms worldwide has posed significant challenges and opportunities for educators (I et al., 2022; Li, 2018; Szelei et al., 2020; Tualaulelei, 2021). In mathematics classrooms, emergent bilinguals (EBs; a.k.a. English learners)—students who are developing proficiency in English or the language of instruction while simultaneously learning complex content—require pedagogical approaches that not only build their mathematical understanding but also support their necessary language development to understand mathematical concepts (García & Kleifgen, 2018). For the dual challenges, the quality of instruction provided to EBs is critical to their academic success. Yet, many mathematics teachers report feeling inadequately prepared to address these students' unique linguistic and cultural needs (Correll, 2016; Lucas et al., 2008).

The intersection of teacher knowledge, quality of instruction, and the educational needs of EBs in mathematics education has garnered increasing attention in recent years. For mathematics teachers of EBs, their capacity to effectively teach EBs extends beyond mathematical content and pedagogy to include the socio-cultural contexts of learners and strategies for integrating language support

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https://doi.org/10.1016/j.jmathb.2025.101281
Received 7 January 2025; Received in revised form 8 August 2025; Accepted 9 August 2025
Available online 19 August 2025

into content instruction (García et al., 2010). However, developing this multifaceted knowledge base is challenging, particularly within the constraints of traditional professional development (PD) models that often fail to address the complex, situated nature of teaching.

We employed collaborative situated professional development (CSPD) to address these gaps. When teachers and researchers engage in co-planning and co-teaching, the PD becomes both collaborative (Holmqvist & Lelinge, 2021)—emphasizing shared learning and reflection—and situated (Lave & Wenger, 1991), directly linked to the teachers' classroom experiences. This type of PD is rooted in teachers' everyday work and involves collaboration between educators and researchers. CSPD can be a powerful approach for effectively enhancing teachers' capacity to teach EBs. CSPD is characterized by its grounding in the specific contexts in which teachers work, allowing them to develop practical skills and knowledge directly applicable to their classrooms through collaboration with researchers. This approach contrasts with traditional, one-size-fits-all PD models by emphasizing the unique challenges and opportunities of teaching EBs in diverse educational settings. This study investigates the impact of CSPD on high school mathematics teachers' instruction for EBs. Prior research has shown that many teachers of EBs tend to reduce the linguistic and cognitive demands of mathematical tasks, often with the intention of making content more linguistically accessible to students still developing English proficiency (de Araujo, 2017; I, 2019). While these adjustments are typically made to support EBs in understanding mathematical concepts or problems, they can inadvertently limit students' opportunities to engage with rich and rigorous mathematical content (I & Chang, 2014; I, 2019; de Araujo, 2017).

To address this issue, our study examines how mathematics teachers engage EBs in meaningful learning using rigorous tasks and improve instructional strategies through teacher-researcher collaboration situated in classrooms. Our research question is, "What instructional changes occur in mathematics teachers' practices for EBs after engaging in CSPD?" This study contributes to the growing body of research on effective professional development models for linguistically and culturally diverse students by focusing on the intersection of teacher education, quality of instruction, teacher-researcher collaboration, and situated learning.

2. Literature review

This literature review explores key approaches to supporting EBs in mathematics education. The first section examines theories and effective teaching strategies promoting language and mathematical learning for EBs. The second section reviews PD programs to enhance teachers' ability to implement these strategies.

2.1. *Effective teaching for emergent bilinguals*

In the U.S., nearly 10 % of public school students are classified as EBs, reflecting the nation's growing linguistic diversity (National Center for Education Statistics, 2024). These students come from a wide variety of backgrounds, including recent immigrants and native-born children from non-English-speaking homes. The term "emergent bilingual," coined by Ofelia García, emphasizes students' full linguistic repertoires and potential for bilingualism, shifting the focus to what they can do and are achieving—becoming bilingual and biliterate—instead of what they cannot (I & Martinez, 2020). Research shows that while many EBs may enter school with lower math achievement, effective support can help them close or even eliminate this gap, especially in schools that recognize and build on their bilingual strengths (Bialystok, 2018).

For teachers working with EBs, "good teaching" and a solid foundation in content knowledge may fall short when applied to linguistically diverse classrooms (Celedón-Pattichis & Ramirez, 2012). Effective instruction for EBs requires more than mastery of subject matter; it demands specialized training that includes an understanding of the linguistic and cultural challenges these students face (García & Kleifgen, 2018). Teachers must develop a critical awareness of EBs' academic and social contexts to tailor their instruction effectively (de Jong & Harper, 2005).

Researchers have employed various theories and pedagogical frameworks aimed at enhancing instructional practices for EBs, including sociocultural theory (Moschkovich, 2002), culturally relevant and sustaining pedagogy (Ladison-Billings, 2014, 1995; Paris & Alim, 2017), and linguistically responsive teaching (LRT) (Lucas & Villegas, 2013). These approaches emphasize the need for instruction that not only addresses academic content but also takes into account the cultural and linguistic assets that EBs bring to the classroom.

Sociocultural theory (Vygotsky, 1978; Moschkovich, 2002) highlights the role of social interaction and language in learning, suggesting that EBs benefit from opportunities to engage in meaningful discourse where their linguistic and cultural identities are recognized and leveraged. Culturally relevant and sustaining pedagogy builds on this by advocating for instructional practices that honor and integrate students' cultural backgrounds and languages as resources, rather than barriers, for learning (Ladison-Billings, 2014; Paris, 2012). This approach promotes academic achievement and aims to sustain students' cultural and linguistic identities within the learning process.

LRT, as described by Lucas & Villegas (2013), involves understanding the specific language demands of academic tasks, particularly in subjects like mathematics, and implementing strategies to support EBs in overcoming these challenges. LRT emphasizes the importance of scaffolding instruction, using visual aids, and incorporating culturally relevant examples to make mathematical concepts more accessible and comprehensible for EBs (Ahn et al., 2015). For example, teachers may break down complex tasks into smaller, manageable steps and provide structured language support to help EBs navigate mathematical language and reasoning. Crucially, integrating students' culture and language within content instruction is not just an additional strategy but a fundamental aspect of improving the quality of mathematics education for EBs. Teachers who view students' linguistic and cultural backgrounds as assets rather than deficits are better equipped to provide meaningful and equitable learning experiences (Cummins, 2000). This culturally and linguistically responsive approach fosters a more inclusive and supportive learning environment, ultimately enhancing EBs' academic success (García & Wei, 2014). For instance, Turner et al. (2013) emphasize the critical role of teachers' knowledge of students' cultural and linguistic backgrounds in shaping instructional practices. When teachers are attuned to the diverse linguistic resources that EBs bring into the classroom, they are more likely to implement strategies that leverage these assets, enhancing both instructional quality and student engagement.

2.2. Teacher PD for EBs

Professional development programs focused on preparing teachers to meet the needs of EBs have been shown to improve instructional effectiveness significantly. Calderón et al. (2011) demonstrated that teachers who engaged in professional development programs integrating language and content instruction were more successful in teaching EBs. These programs often emphasize LRT, culturally responsive pedagogy, and differentiated instruction based on students' varying language proficiency levels. Professional development fosters more equitable and effective mathematics instruction by equipping teachers with the tools and knowledge necessary to support EBs. Teachers must be prepared to integrate language support with mathematical content and employ strategies that recognize and leverage the linguistic resources of EBs. Professional development focusing on these areas is essential for improving both instructional practices and learning outcomes for EBs, creating a more inclusive and effective learning environment.

Research identifies several characteristics that make PD effective across different educational contexts, such as incorporating active learning and allowing teachers to engage with the content through discussions, practice, and reflection. The duration and intensity of PD also matter; sustained and intensive PD is more likely to lead to changes in instructional practice (Garet et al., 2001). Furthermore, PD should be aligned with teachers' ongoing work and the broader school context, promoting coherence and integration into daily practice. Regarding PD for teaching EBs, Li's (2018) analysis found that most programs focus on cultural diversity rather than language and linguistic challenges. They also reveal several challenges in integrating language and linguistic diversity into teacher education: a lack of faculty expertise in EBs, programmatic constraints, and minimal policy support.

Tualaulelei's (2021) ethnographic study of Samoan students in Australia revealed that teachers encountering significant cultural differences in highly diverse classrooms often lacked sufficient PD opportunities for teaching culturally and linguistically diverse students. The study also identified a shortage of both professional and human resources in this area. Similarly, Szelei et al. (2020) reported tensions between existing PD offerings and the actual needs of teachers in Portuguese schools for supporting EBs. Teachers expressed a need for more targeted information, greater collaboration, and enhanced organizational support in their PD. From a critical multicultural perspective, Szelei and colleagues (2020) found that PD on cultural diversity often lacked coherence, with conflicting priorities, fragmented teacher collaboration, and limited involvement of students and the broader community in shaping PD efforts.

Collaborative professional development (CPD, Holmqvist & Lelinge, 2021) can address and respond to these needs by involving educators working together to improve their instructional practices through shared learning experiences and peer interactions, enabling them to engage in joint problem-solving, reflective practice, and the co-construction of knowledge (Vescio et al., 2008). CPD often includes lesson study, peer observations, and professional learning communities (PLCs), where teachers collaboratively plan, observe, and discuss their teaching practices. Research indicates that CPD can significantly enhance teachers' instructional practices by providing opportunities for shared expertise and mutual support. For instance, in mathematics education, the collaborative model of a teacher and a researcher positively impacts the teacher's pedagogical beliefs and practices on students' mathematical performance (Jung & Brady, 2016). However, the review of Holmqvist and Lelinge (2021) found that few studies had a design where the teachers and researchers worked collaboratively to enhance the teaching practices.

Situated PD is rooted in sociocultural theories of learning, which emphasize the importance of context, social interaction, and cultural tools in the learning process (Lave & Wenger, 1991). The theories suggest that learning is most effective when it occurs within authentic environments where knowledge is applied, allowing teachers to engage in practices that are directly relevant to their instructional contexts; for teachers of EBs, situated PD offers the opportunity to develop strategies that are responsive to the linguistic and cultural diversity of their students, thereby enhancing the quality of instruction. Situated PD is often grounded in the context of teachers' actual classroom practices, focusing on real-world teaching challenges and applications. This approach often includes components such as co-planning and co-teaching with experts, analyzing student work within the classroom setting, and adapting PD activities to fit the specific needs of teachers and students. By embedding PD within the classroom environment, teachers can immediately apply new strategies and receive direct feedback on their implementation (Cohen & Hill, 2001). Research supports the effectiveness of situated PD in improving teaching practices and outcomes for EBs in mathematics education (e.g., I et al., 2020). Aguirre and del Rosario Zavala (2013) found that when teachers participated in situated PD within their specific classroom contexts, they were more successful in integrating language and content instruction. This was particularly evident in their ability to design and implement mathematics lessons that were both linguistically accessible and cognitively demanding, leading to improved student engagement and achievement.

By combining the collaborative aspects of PD with the contextual focus of situated learning, we conducted CSPD with high school mathematics teachers of EBs for this study. CSPD represents a synthesis of the strengths of both collaborative and situated PD approaches.

3. Methods

This section outlines the methodology employed in the study, including details about participants, context, and the CSPD design.

3.1. Participants and context

Our study involved seven high school mathematics teachers with varying teaching experiences (see Table 1). Six of them are native English speakers, and four are bilinguals with international backgrounds (e.g., study abroad, immigration, or international marriage). The teachers taught either Algebra 1 (four teachers) or Pre-Algebra (three teachers) in classrooms where all students were identified as EBs by the school district. The students in these EB-only mathematics classrooms represented diverse linguistic, racial, geographical, and cultural backgrounds. Over ten home languages were spoken by the 83 students from these classrooms who voluntarily participated in this study.

All teachers possessed a mathematics license and had not received any math-specific EB-related PD until participating in this project, but their knowledge backgrounds varied. For example, Ellie was working on her doctoral degree in educational psychology with a focus on social-emotional learning and ESL endorsement. She studied abroad in several different countries and taught English in Spain and Spanish at a U.S. school. Due to her ability to speak Spanish, her school placed many Spanish-speaking EB students at a low English level in her class.

We collected data over three years in collaboration with four high schools from a large urban district in the Midwest, USA, where more than 100 languages are spoken, and over 20 % of students are EBs. The district offers two levels of K-8 mathematics courses, including Pre-Algebra, specifically designed for high school students with interrupted formal education, such as refugees. All Algebra 1 classes, including EB-only Algebra 1, implemented the Illustrative Mathematics (IM) curriculum, while two Pre-Algebra teachers used the middle school IM curriculum as a resource to guide their instruction and one Pre-Algebra teacher used her own curriculum.

As shown in Fig. 1, the majority of participating students were classified at the Emerging (beginner) or Progressing (intermediate) levels of English proficiency. Only two students achieved a Proficient designation. Some students' scores are missing due to absences or other reasons for not completing the assessment. The duration of students' residency in the USA varied widely, ranging from less than one year to over ten years, including one student who was born in the U.S. For instance, in Year 2, the median length of time students had lived in the U.S. was three years, with a range from one year to sixteen years (i.e., their entire lives).

3.2. Collaborative situated professional development design

During our CSPD, each of the seven teachers actively participated for one school year. The program involved co-planning, co-teaching, and co-reflection sessions with the research team approximately once a month. Together, each teacher and the research team collaboratively designed and implemented seven lessons per classroom over the academic year. Some lessons were used for multiple classrooms with necessary modifications. In this collaborative process, the researcher's role was to provide resources and knowledge about research-based strategies tailored to EBs. Meanwhile, the teacher played a crucial role as the decision-maker on problem difficulty level and familiar context, drawing on their understanding of students' existing knowledge and interests. The co-planning session occurred approximately one week before the co-teaching session, allowing the teacher to make detailed predictions about student learning outcomes. The teacher and a researcher (first author), took turns leading different segments of the co-taught lessons and actively engaged with students throughout the sessions. The first author, currently a mathematics education researcher and teacher educator at a four-year university, is a former secondary mathematics teacher in the USA and a former EB herself, having immigrated to the U.S. as an adult. The second author is a monolingual English-speaking doctoral candidate and a former high school mathematics teacher who was actively involved in the co-planning and co-reflection sessions, though did not co-teach. The third author, an international student bilingual in English and Nepali with no formal teaching experience, contributed by offering bilingual insights during some co-teaching sessions and facilitating select co-reflection discussions. Over time, we intentionally shifted the distribution of roles, gradually decreasing the researcher's instructional involvement and increasing the teacher's leadership and agency. After each co-teaching session, the teacher and research team engaged in structured debriefs to reflect on the lesson and analyze student learning. These co-reflection sessions also served as a space to collaboratively set instructional goals and identify areas for improvement in preparation for subsequent lessons.

Table 1
Teacher Participants Backgrounds.

Year	Name	Subject	Teaching experiences (years)	EB-only class experiences (years)	Teacher languages	Student languages
1	Eva	EB-only Algebra 1	3 (in the US), 15 (in her home country)	3	Spanish, English	Spanish, Kunama, Swahili, French, Vietnamese
2	Nina	EB-only Algebra 1	3	1	English, Bengali	Spanish, Swahili, Arabic, Karen, Kunama, Vietnamese, French
	Hannah		25 +	7–8	English	Spanish, Swahili, Burmese, Karen, Karenni, Kunama, French
	Richard		23	6	English	Spanish, Swahili, Pashto, Arabic, Somali
3	Ellie	EB-only Pre-algebra	5	2	Spanish, English	Spanish, Persian, Swahili, Arabic
	Harry		9	9	English	Spanish, Swahili, Karen, Kunama, Sundanese, Indonesian, Pashto
	Roberto		3	1	Spanish, English	Spanish, Arabic, French, Swahili, Persian, Tigrinya, Pashto

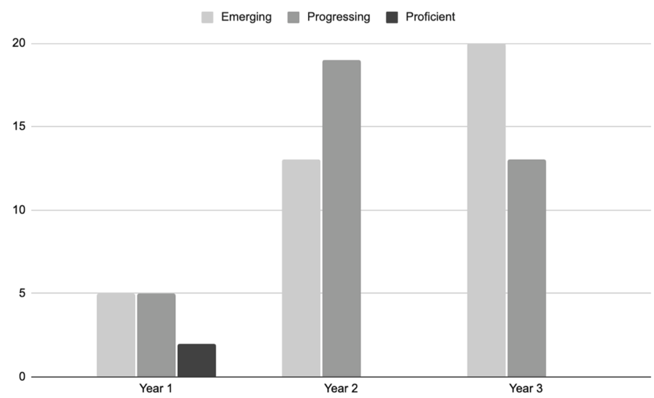


Fig. 1. Participating students' ELPA 21 scores by years.

We began by observing each teacher's independent lesson (pre-lesson) to establish a baseline. This was followed by seven cycles of co-planning, co-teaching, and co-reflection with each teacher. To assess the impact of CSPD, each teacher then taught another independent lesson (post-lesson) after the collaborative phase, without real-time pedagogical coaching. These post-lessons demonstrated the extent to which teachers internalized and applied strategies developed during CSPD. The professional development emphasized modeling asset-based perspectives rather than directly correcting deficit views, supporting teachers in independently transferring these approaches into their practice.

3.3. Curriculum framework: 5-Act Task

Each co-teaching lesson was designed to actively engage EBs in rigorous mathematics and real-life contexts using a 5-Act Task structure (I et al., 2020), a modified version of the 3-Act Task proposed by Meyer (2011). The 5-Act Task consists of the following acts:

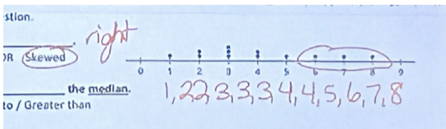
- Act 0 (Assessing/Scaffolding): We assess and scaffold the language, mathematics, and context required to understand the main story in Act 1. EBs explore language within rich, authentic situations, preparing them for the mathematical concepts introduced in the lesson.
- Act 1 (Problem Proposing): Based on a story with conflicts, Act 1 presents the problem to students, often with visuals and multimedia. It encourages curiosity and sets the stage for mathematical exploration.
- Act 2 (Collect Information and Select Tools/Strategies): Students collect relevant information and choose appropriate tools or strategies to tackle the problem.
- Act 3 (Solve Problems and Resolve Conflicts): Act 3 involves solving the problem and resolving any conflicts that arise during the process. During this time, EBs have opportunities to build their own models without teacher help before teachers provide guidance.
- Act 4 (Collective Communication of Solutions/Thinking Process): Act 4 emphasizes students' collective communication on mathematics. EBs share their ideas, demonstrate understanding, and take pride in their linguistic and mathematical abilities in a safe environment.

Moreover, we employed multiple strategies to support EBs in the 5-Act Task framework. The strategies used in nearly every co-teaching session include (1) delaying answers until students have sufficiently explored the problems (in Act 3), (2) encouraging students to share observations and wonderings about the story before diving into mathematical solutions (Notice and Wonder) (in Act 0 or 1), (3) group activities with collective note-taking (in Act 1 or 3), (4) integrating familiar real-life contexts and multimedia (e.g., images, video clips, and animations) (in Act 1), (5) utilizing students' native languages through translanguaging approaches (in Acts 1, 3, & 4), and (6) mathematics journaling (in Act 4).

Notice and Wonder (e.g., Ray-Riek, 2013) is a strategy in which students are presented with a visual and are asked to analyze it with a mathematical lens. Students make mathematical observations (i.e., Notice) and pose mathematical questions (i.e., Wonder) about the situation depicted in the picture or video. We encouraged the EBs to start with non-mathematical observations and questions for additional contextual and linguistic support. Ideally, the students ask a mathematically motivated question relevant to the lesson's mathematical concept, in which case the Notice and Wonder activity can transition to the main learning activity. Otherwise, the teacher may need to guide students' questioning toward the day's concept.

In each co-teaching lesson, we insisted that students work in groups for mathematics activities. We often began with group discussion strategies like Think-Pair-Share (Kaddoura, 2013), which encourages students to write down their thoughts and discuss ideas with peers and then as a class. This group discussion allowed students to practice problem-solving, exchange strategies, and communicate about mathematics (Zahner, 2012). Furthermore, due to the collaborative nature of group work, we utilized collective note-taking, providing each group with a large piece of paper and different-colored markers for each member. This approach facilitated the identification of individual contributions while encouraging students to learn from one another and provide feedback as they worked toward completing the activity. It also enabled teachers to recognize students' progress and intervene when assistance was

Table 2
Descriptions of 12 MQI and QLDT codes with examples.

Code name	EB-focused description	Examples
MQI		
Students provide explanations	Students explain a mathematical or non-mathematical idea, even if incomplete or inaccurate, using writing, gestures, or their first language.	T: Cual trabajo es mejor para ti? [Which job is best for you?] S: Ninguno [none] T: No, por qué? [no, why?] S: Porque no me interesa trabajar en esto. [because I'm not interested in working on this]
Student mathematical questioning and reasoning	Students engage in mathematical thinking with features of essential mathematical practices, such as reasoning and questioning about math in oral or written forms.	[The task requires students to label, using a variable, the length of two sides of a canvas of one side is 2 in. longer than the other.] S: So why is it a square over there? You said rectangle. Why is it square over there? (High if students' questioning and reasoning characterizes the segment)
Students communicate about the mathematics of the segment	Students communicate their mathematical ideas in whole-class or small-group settings. It includes simple answers to the teacher's questions or group discussions.	T: That is going to be −12 and I don't want −12. So what else I need to do? S: We can do negative four times negative three. T: Both negative. Okay, good. And that is going to be positive 12. So, I don't need to put the plus. What else? S: Negative six by two. (High if substantive students' contributions characterize the segment)
Task cognitive demand	Students engage in tasks in which they think cognitively and reason about mathematics, using the cognitive demand level (Stein et al. 2009). Teachers' heavy scaffolding may reduce the cognitive level.	[Provides an extension activity to create a quadratic equation to model a new flight path of a bird in the Angry Birds game.] T: … Then you have a challenge as a group to see if you can make a new path of a bird. Like a fourth one that we didn't see. (High if students engage with cognitively demanding tasks throughout the segment)
Students work with contextualized problems	Students work with a contextualized problem (e.g., story problems, real-world applications, experiments), including any word problem, regardless of the context or metaphorical use of a real-life example.	[Provides an activity which utilizes a recipe as a metaphor for the procedure of solving quadratic equations] T: Even if he's not a good cook, if he follows the steps, the pancakes will still turn out good. That's what's great about a recipe. Same with this. Some of you might think, "I'm not great at math." It's okay. If we follow the steps, we can do this. (Low because the implemented task did not ask students' reasoning)
QLDT		
Connections with mathematics and students' life experiences	The teacher or students make connections between life experiences to mathematics.	[When explaining the importance of finding the correct frequency interval in a histogram] T: My question is, where does the number 20 [20 appeared in two intervals] go? Because there's two places I see it. It's like, if I looked at [Angela's]schedule, and it said, second hour she's in math, and second hour she's in art, where are we supposed to be? (High if this characterizes the segment)
Connections with mathematics and students' prior knowledge	The teacher activates students' prior knowledge by explicitly referencing skills learned in a previous lesson or grade.	T: What is the "mean"? Anyone remember? So, in both of these [pointing to two distribution graphs on the board]], are they the same or different? (High if this characterizes the segment)
Connections of mathematics with language	The teacher connects language with mathematical representations such as pictures, tables, graphs, and mathematical symbols.	
Meaning and multiple meanings of words	The teacher or students communicate meaning by using synonyms, gestures, drawings, cognates, or translations of students' first language.	T: We got 8.5. This is how long it takes her to get to school on the average day. Okay? It means that on average, or typically… You guys know the word, typical? …Like usually. Like most of the time it will take 8.5 min to get to school. (High if this appears extendedly)
Use of visual aids or support	The teacher uses visual supports in classroom conversations beyond the static nature of the blackboard.	[Played a video clip about Angry Bird and followed it with a notice and wonder to assess student readiness] T: What is this point [gesture] that would be way up here called? S: Maximum vertex. T: A maximum vertex (High if the visual use clearly enhanced math reasoning)
Record of written essential ideas, concepts, representations, and/or words on the board	The teacher and/or students display/record/use and retain written important information on the board.	T: We're talking about using our shape. Is it symmetric? Is it skewed? If you forget those words, we have our word wall. [wall in the back of the room with different distribution shapes drawn and labeled] (High if the record sustained throughout the segment)
Discussion of students' mathematical writing	The teacher or students use students' mathematical works publicly or have a private discussion of written work in small groups or individual students.	[Using Spanish, the teacher interacted with one student using the student's written work] S: So, I put these two? T: Why? You can use this [pointing students' work]. How many numbers? S: Four. T: Three, only three. Because fourteen numbers cross one of the two sides, until this [pointing] S: Two. Here I divide by two everything. (High if it sustains throughout the segment)

needed.

Finally, we aimed to support EBs using their full linguistic repertoire, not just English, by incorporating translanguaging strategies into each co-teaching lesson (Yi et al., 2024; García & Wei, 2014). Translanguaging goes beyond simple translation; it allows students to communicate flexibly using multiple languages. In the co-teaching lessons, we displayed the main problem of the day translated into various students' native languages and invited EBs to read it aloud in their languages. We encouraged EBs to present their ideas in their preferred languages while the audience used their written work and gestures to follow along. Additionally, we allowed students to respond to written prompts in their chosen languages because we recognized that written responses require significant time and cognitive effort, as EBs must process and translate ideas beyond the mathematical content of the task.

3.4. Data collection and analysis

The primary data for this study consists of 45-minute lessons observed and videotaped before and after the CSPD. All sessions were transcribed, with Spanish translations added as needed.

For coding purposes, two coders—the first and second authors—analyzed each independent lesson using video clips and transcripts. We employed the Mathematical Quality of Instruction (MQI; Hill et al., 2005) and the Quality of Linguistically Diverse Teaching (QLDT; Sorto et al., 2018) frameworks to examine the teachers' instructional approaches. The QLDT codes were developed to supplement the MQI, so the EB-related codes, such as contextualized tasks, in the MQI are not duplicated within the QLDT framework. The class recordings were divided into 7.5-minute segments, following the MQI direction and allowing us to assess various instructional actions and student communications. The MQI framework uses 7.5-minute segments, based on raters' perceptions that longer segments were more cognitively demanding to code, while shorter segments required more time and resources (Kraft & Hill, 2019). We modified the QLDT codes by splitting the first code, 'Connections of mathematics with students' life experiences and prior knowledge,' into two separate codes—one for connections with students' life experiences and another for connections with their prior mathematical knowledge because we believe that each aspect holds distinct significance for teaching EBs. Given our study's primary focus on teachers' improvement in maintaining cognitive demand in mathematics and language and implementing linguistically responsive teaching strategies throughout the school year, we chose to use one domain of the MQI codes, Common Core Aligned Student Practices, which includes five codes. This resulted in the 12 codes described in Table 2.

We assessed the assigned codes for each video segment using a scale from 0 to 3 (0 = Not Present, 1 = Low, 2 = Mid, 3 = High). When two coders assigned different scores, they discussed the discrepancies until reaching a consensus. For each lesson, we first calculated the mean score for each code across the six 7.5-minute video segments from both the pre- and post-lessons. However, given the natural variability in teaching, it is unrealistic to expect uniform instructional quality across all segments. To account for this, we also recorded the maximum score for each code, as mean values can sometimes obscure meaningful instructional moments. For example, in Nina's post-lesson, the "Students Provide Explanations" code received scores of 2, 1, 1, 1, 2, and 2 across the six segments, resulting in a mean of 1.5 and a maximum of 2. This dual-metric approach allowed us to better capture shifts in mathematics instruction between the pre- and post-CSPD lessons.

To assess the impact of the CSPD, we calculated the difference between post- and pre-CSPD scores for each teacher on both the mean and maximum values for each code. For example, Nina's post-CSPD mean score for the "Students Provide Explanations" code was 1.5, compared to a pre-CSPD score of 0.833, resulting in a gain of 0.667. These per-teacher difference scores for each code—both mean and maximum—comprised the dataset used to evaluate whether teachers demonstrated statistically significant improvements in instructional practices following the CSPD.

We first tested the normality of the difference scores using the Shapiro-Wilk test. Depending on the distribution, we employed

Table 3
All Teachers' Score Differences in MQI and QLDT Codes.

Code	Average of the differences in mean scores (# of teachers who improved)	Average of the difference in maximum scores (# of teachers who improved)
MQI_Common Core Aligned Student Practices		
Students provide explanations	0.69 (7)	0.29 (2)
Student mathematical questioning and reasoning	0.48 (5)	0.71 (5)
Students communicate about the mathematics of the segment	0.41 (4)	0.29 (3)
Task cognitive demand	0.83 (6)	0.86 (5)
Students work with contextualized problems	0.52 (4)	0.71 (3)
Quality of Linguistically Diverse Teaching		
Connections with mathematics and students' life experiences	0.64 (4)	0.71 (4)
Connections with mathematics and students' prior knowledge	0.10 (3)	0.71 (3)
Connections of mathematics with language	0.14 (4)	0.29 (2)
Meaning and multiple meanings of words	0.17 (3)	0.43 (3)
Use of visual aids or support	0.76 (6)	1.29 (5)
Record of written essential ideas, concepts, representations, and/or words on the board	0.71 (6)	0.43 (2)
Discussion of students' mathematical writing	0.14 (5)	0 (3)

either a paired *t*-test for normally distributed data or the Wilcoxon signed-rank test for non-normal data. To enrich our interpretation of the quantitative results, we incorporated qualitative evidence from classroom recordings. These examples illustrate how teachers adapted their instructional practices over time, particularly in relation to codes that showed statistically significant improvement, providing a more comprehensive understanding of the shifts in instructional quality.

4. Findings

Table 3 presents the changes in all teachers' mean and maximum scores across the 12 codes. Specifically, for each code, the average values of mean and maximum score differences from pre- to post-CSPD lessons across all seven teachers were calculated and examined. Overall, all codes showed increases from the pre- to post-CSPD lessons, though the degree of improvement varied by code. For instance, the QLDT code "Connections with Mathematics and Students' Prior Knowledge" exhibited the smallest increase in mean, with only three teachers showing gains. In our modified coding manual, this code specifically captures connections to previous lessons or to mathematical concepts taught in prior years. Because the definition of this code is relatively narrow and the CSPD sessions did not explicitly emphasize this instructional move, it is unsurprising that fewer teachers demonstrated growth in this area.

For some codes, six or all of the seven teachers showed large improvements in mean scores. Specifically, large gains between pre and post-CSPD lessons were evident in MQI codes: "Students Provide Explanations" (0.69) and "Task Cognitive Demand" (0.83), as well as in QLDT codes: "Use of Visual Aids or Support" (0.76) and "Record of Written Essential Ideas, Concepts, Representations, and/or Words on the Board" (0.71). Instructional practices relevant to these codes were highly emphasized within the 5-Act Task structure during the CSPD, such as tasks with real-life context and multimedia, dedicated spaces for student sharing in Act 4, intentional delay of teacher help, and use of open-ended learning activities like Notice and Wonder. We observed that teachers readily adopted these emphasized strategies in their post-lessons, which likely contributed to the observed increases in these scores.

For many codes, we observed more teachers improving their mean scores compared to their maximum scores. The specific example of code "Student Provide Explanations" highlights this trend, where the mean difference was 0.69 with all teachers showing improvement, while the maximum difference was 0.29 with only two teachers improving and five teachers maintaining the same maximum score. This pattern may suggest that the CSPD was effective in raising the consistent application of these instructional practices throughout the post-lesson segments for many teachers, even if it did not always translate to higher peaks of instructional practices. While achieving new heights of peak performance (reflected in maximum scores) is one measure of growth, the broader rise in average performance (reflected in mean scores) demonstrates a foundational and widespread improvement in teaching quality.

Conversely, for the code "Connections with Mathematics and Students' Prior Knowledge," the average maximum score difference was notably higher (0.71) compared to the small growth in its mean counterpart (0.10). This discrepancy suggests that while the practice of connecting mathematical concepts to students' prior knowledge may not have been consistently embedded in the post-lessons across all segments, the teachers were able to implement it at a higher proficiency during specific moments of the lesson. These 'peak' performances might indicate a developing ability that was not fully routinized in lessons within the CSPD timeframe.

The differences in the mean pre- and post-scores for the seven participating teachers show statistically significant improvements in two codes: "Students Provide Explanations" and "Task Cognitive Demand." Similarly, the changes in the maximum pre- and post-CSPD scores reveal statistically significant enhancements in three codes: "Student Mathematical Questioning and Reasoning," "Task Cognitive Demand," and "Use of Visual Aids or Support." A summary of the results for the codes with significant changes is provided in Table 4.

These results suggest that teachers made measurable gains in several key instructional practices. However, the lack of significant change in the other seven codes may reflect the complexity of instructional improvement, limited sensitivity of the codes, or variability in the CSPD's focus. Additionally, differences in teachers' prior experiences and knowledge likely contributed to inconsistent shifts across participants. While some teachers showed substantial improvement, the overall shift was not statistically significant due to uneven progress among the seven teachers. Despite this, the significant improvements in multiple codes indicate that the CSPD program positively impacted specific teaching dimensions, even if changes were not uniform across all areas.

In the following sections, we present detailed examples for each code identified as showing significant improvement. We begin by outlining the results for each teacher within these key codes, highlighting how much individual teachers shifted. To deepen our understanding, we also provide qualitative examples that illustrate these outcomes, which serve to contextualize the quantitative data, providing a richer, more nuanced picture of the observed instructional changes and their impact on student learning.

Table 4
All teachers' results in the codes with statistically significant changes.

Code	Average of pre-lesson mean scores	Average of post-lesson mean scores	Difference (Post-Pre)	Statistical significance test results
Students provide explanations	0.4762	1.1667	0.6905	T-test, $p = 0.0085$
Task cognitive demand	0.6667	1.5000	0.8333	T-test, $p = 0.0098$
Code	Average of pre-lesson max scores	Average of post-lesson max scores	Difference (Post-Pre)	Statistical significance test results
Student mathematical questioning and reasoning	1.2857	2.0000	0.7143	Wilcoxon Test, $p = 0.0253$
Task cognitive demand	1.4286	2.2857	0.8571	T-test, $p = 0.0166$
Use of visual aids or support	1.0000	2.2857	1.2857	T-test, $p = 0.0488$

4.1. Students provide explanations

Among the codes exhibiting significant changes, the "Students Provide Explanations" was the only code that demonstrated an increase for all teachers from pre- and post-CSPD scores. Fig. 3 displays the varied increasing shifts of the teachers, and Hannah demonstrated the most growth, from 0.83 to 2.33. We observed that the teaching experience length is not related to the gain in this code because the experienced teachers, Hannah (25 years) and Richard (23 years), showed very different growth. Similarly, while a teacher's bilingualism can be helpful in some cases, it is not an absolute necessity for encouraging students' mathematical explanations. The growth of bilingual teachers—Roberto, Ellie, and Eva—was not noticeably different from that of the monolingual teachers. In fact, Hannah, a monolingual English speaker, showed the largest improvement.

While the students in Hannah's class provided explanations frequently during both pre- and post-CSPD lessons on variability, their explanations were mainly brief in the pre-CSPD lesson, as shown in the following excerpt:

Teacher: The mean changed. What's going to happen to our variability if we add this big number?

Students: Spread will be more.

Teacher: Spread's going to be more. What's going to change more? The IQR or the standard deviation?

Students: Standard Deviation.

Teacher: Good. Standard Deviation goes with the mean.

In this excerpt, the teacher (Hannah) switched her question from an open-ended format to a multiple-choice format with two options (IQR or standard deviation), prompting students to respond with only one or two words. In Hannah's post-CSPD lesson about quadratic equations, the students were verbally present for more instructional time, explaining their thought processes to the teacher and students, as the following conversation shows.

Student: So, in this case, if we time this itself, so…we have to put this two times, and then we want to distribute.

Teacher: You can. That's not what I would do. But you can, you will get the same answer.

Student: So, you think it would be better if we distribute the negative one first?

Teacher: You can't distribute the negative one first because the order of operations…Parentheses, exponents, then multiply. So, you have to do this exponent before you can do any distribute.

Student: But what if it was just that?

Teacher: What if it was that?

Student: Without the negative one, we cannot use the square root because, by this number…

Teacher: But you're thinking square root, right?

Student: Yeah, I think it's square root.

Teacher: Is there something we can do to get rid of this?

Student: Inverse operation?

Teacher: Inverse operation. That says multiply. What's the inverse of multiply?

Student: Divide.

Teacher: So, you could do that too, because if you divide both sides, what happens to that? What's negative divided by negative?

Student: It's positive. All right.

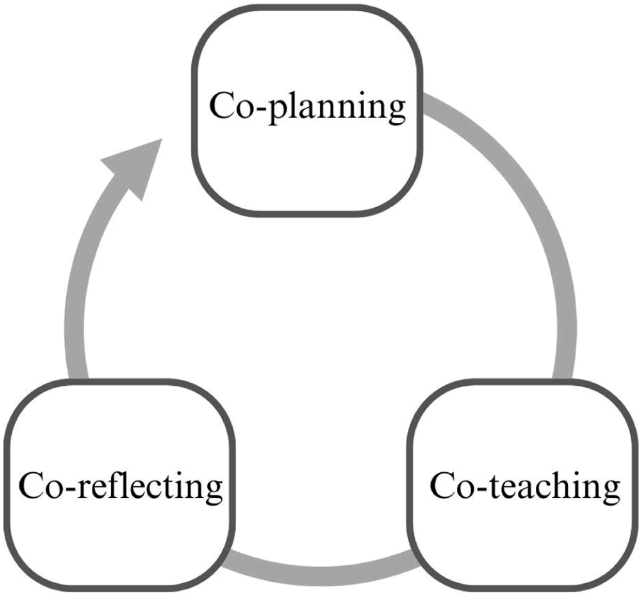


Fig. 2. CSPD model cycle.

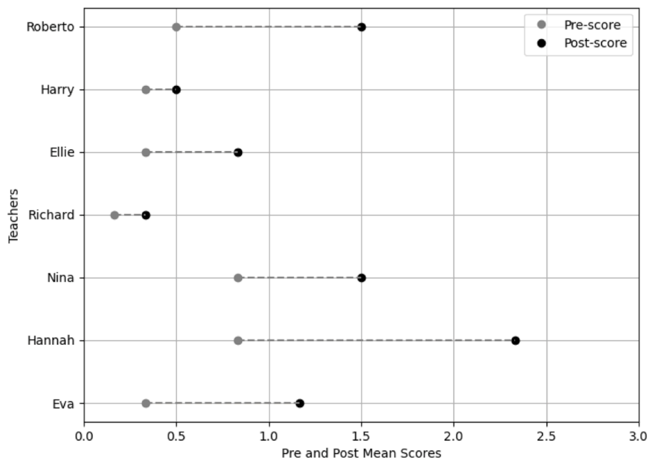


Fig. 3. Mean pre- and post-CSPD scores for code "Students Provide Explanations".

In this excerpt, the student wanted to apply a square root to solve a quadratic equation but could not due to the -1 coefficient. The teacher guided the student on how to remove the negative sign. During this conversation, they discussed multiple mathematical ideas involving square roots, distribution, the order of operations, inverse operations, and negative signs.

Another example of student explanation comes from one of Roberto's lessons delivered in the second half of the CSPD cycle. In his pre-CSPD lessons, Roberto primarily focused on step-by-step algorithms, resulting in one-way, lecture-style instruction. In contrast, this lesson—centered on identifying the solution to a system of equations—featured more active interactions between Roberto and his students, with dialogue taking place in Spanish.

> Student 1: *Yo dije que no importa la hora que trabajes, Target va a ser el que mejor paga* [I said that no matter what time you work, Target is going to be the one that pays the best.]
>
> Teacher: *Entonces, ¿tú crees que… cómo? ¿Cómo? Dime otra vez.* [So, you think that… How? How? Tell me again.]
>
> Student 1: Yo dije que no importa la hora que trabajes, Target paga mejor porque… trabajas dos horas y, en una hora, yo gano más. Entonces, si trabajas una hora, vas a ganar 10, pero si yo trabajo una hora, voy a ganar 20. [I said that no matter what time you work, Target pays better because… you work two hours and, in one hour, I earn more. So, if you work one hour, you'll earn 10, but if I work one hour, I'll earn 20.] En el segundo, el segundo… siempre vas a ganar más en el segundo porque… [In the second one, the second… you're always going to earn more in the second because.]
>
> Teacher: Entonces, piensa en lo que dijo David: si nomás trabajas una hora, es mejor trabajar en el primero, pero si trabajas más, como seis o siete horas o algo así…[So, think about what David said: if you only work one hour, it's better to work at the first one, but if you work more, like six or seven hours or something like that…]

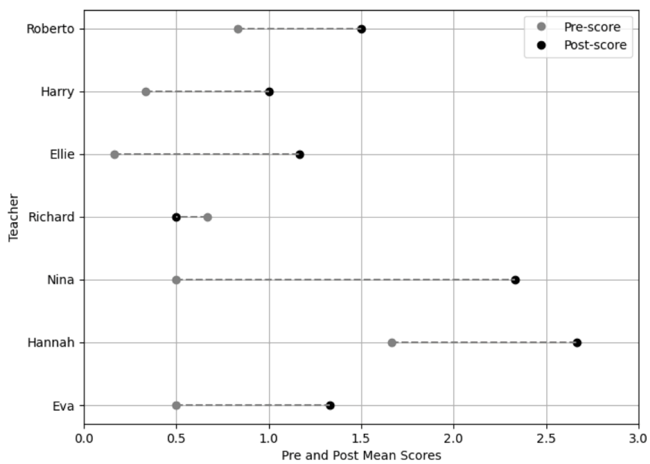


Fig. 4. Mean scores in pre- and post-CSPD for code "Task Cognitive Demand".

Student 2: Igual, si trabajas como siete horas en el primero, igual seguirás haciendo menos dinero. En el segundo, si trabajas siete horas, vas a hacer más dinero. [Even if you work like seven hours at the first one, you'll still make less money. At the second one, if you work seven hours, you'll make more money.]

In this excerpt, the student explained that working at the second store would result in a higher overall wage, taking into account both the hourly pay and the sign-on bonus, which can be solved by a system of linear equations or inequalities. Through a conversation in Spanish with the teacher, the students were able to articulate their mathematical reasoning more fully and clearly.

Despite their differing backgrounds in bilingualism and teaching experience—one being a novice teacher (Roberto) and the other a seasoned veteran (Hannah)—both teachers demonstrated meaningful improvement in encouraging and valuing students' mathematical explanations. This suggests that participation in the CSPD program supported instructional growth across varied teacher profiles, regardless of prior experience or language background.

4.2. *Task cognitive demand*

Six teachers had positive shifts in "Task Cognitive Demand" when comparing the mean values. As shown in Fig. 4, Nina showed the largest improvement in mean scores for "Task Cognitive Demand" from pre- to post-CSPD lessons (0.50–2.33). One possible explanation for the observed improvement is that this was her first time teaching a mathematics course composed exclusively of EBs. As an early-career teacher, she may have been particularly receptive to learning and growth through the CSPD process. Research suggests that novice teachers often demonstrate greater openness to new pedagogical approaches, particularly when they align with perceived instructional challenges (Ainley & Carstens, 2018). A central focus of the co-planning sessions across all teachers was the selection and adaptation of cognitively demanding mathematical tasks that promote conceptual understanding—an instructional approach shown to support equitable learning opportunities for multilingual learners (Moschkovich, 2013). In contrast, only one teacher (Richard) showed a decrease in scores from pre- to post-CSPD. This may be attributed to the highly procedural nature of his post-CSPD lesson, which focused on practicing algorithms for solving quadratic equations. Procedural lessons often provide limited opportunities for student reasoning, explanation, or language use (Boaler & Staples, 2008), which are key dimensions in our coding framework and central to high-quality instruction for EBs. It should also be noted that the teachers' initial stages varied from 0.2 (Ellie) to 1.7 (Hannah), which may be influenced by their previous teaching experiences.

This code also appears significant in the maximum score analysis. Fig. 5 illustrates the changes in maximum scores for the seven teachers between pre- and post-CSPD. Of the seven teachers, five exhibited positive changes, while two showed no change. The teacher with the greatest increase in mean scores (Nina) also had the most considerable improvement in maximum scores, rising from 1 (low) to 3 (high). In contrast, the teacher who experienced a decrease in mean scores (Richard) maintained the same maximum scores.

Considering that most teachers used the problem-based curriculum, which includes many cognitively demanding tasks, we observed that the teachers tended to provide more scaffolding than was necessary, limiting the richness of the mathematics and student agency before participating in the CSPD. One example of over-scaffolding is the handout provided for students to work through the concept of the day, finding outliers, as shown in Fig. 6. For instance, in the pre-CSPD lesson, Nina guided students through the problem without allowing them to deeply explore the mathematical concepts or share what they had learned. Nina quickly moved on without seeking input from the student, saying, "That's how we draw it, right? Then we want to find how big our box was. We subtracted what

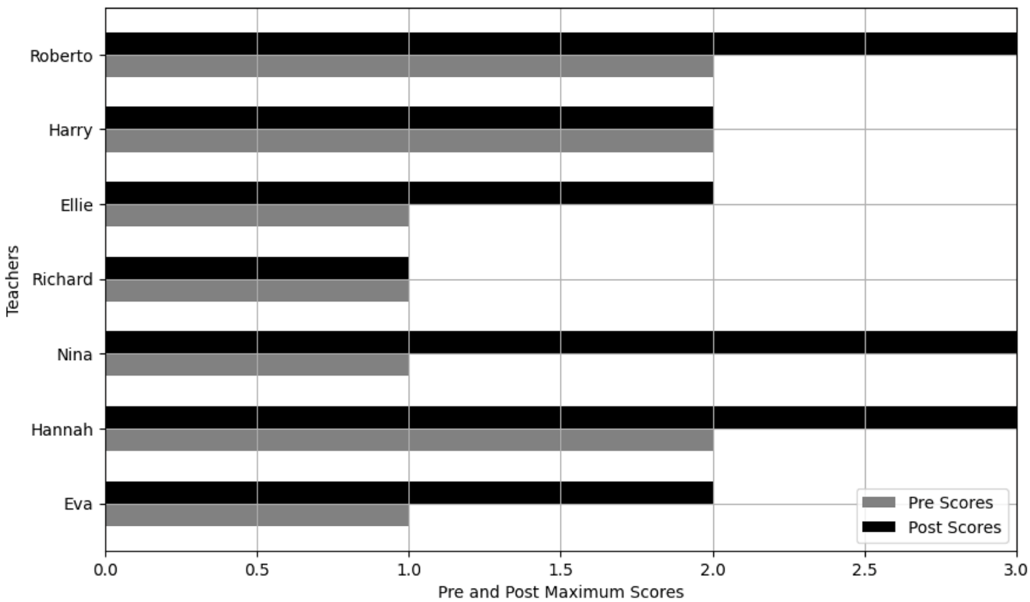


Fig. 5. Maximum scores in pre- and post-CSPD for code "Task Cognitive Demand".

from one; we did Q3 minus Q1 to find out how big our box, that is our IQR, the size of that box. So, to find our IQR or interquartile range in this case, I don't have a box, but I have the things I need to find it."

In the post-CSPD lesson, Nina created space and time for students to grapple with important concepts rather than over-scaffolding them before students explore or share knowledge. To introduce perfect squares, she displayed several expressions of perfect squares on the board and asked students to discuss what defined a perfect square. "What are some of the observations you guys noticed what might have something that gives them all the same title of perfect square? [call on one student] Could you share some of your notes that you had there?" The students proceeded to discuss what they noticed with encouragement from the teacher. Guiding the lesson, the teacher asks, "What is happening? If I was to use my box method (area model for multiplying and factoring polynomials) to try to rewrite this. What would I put on my box method? […] What would be on the outside?" (Student: "*x* square.") The teacher and students continued down this path, using what they knew about factoring expressions to reveal two equivalent binomials before further narrowing the definition of a perfect square. (Student: "It's going to be the same" [indicating the two factors found using the area model are the same.]) "So each of these, we're taking what? Same height and width [of the rectangle used for the area model]. Are those the same height and width? What about those ones? Did anyone figure out how wide, tall they would be? Give it a try." Here, the teacher demonstrates skilled support without depriving her students of mathematical rigor.

Another example of over-scaffolding, which subsequently reduced the cognitive demand of the task, occurred during Hannah's pre-CSPD lesson on measures of central tendency. In this instance, she guided a student through the process of identifying the median in an ordered data set by explicitly directing each step:

> "Cross off. Cross off the short. Cross off the tall—correctly [student follows instructions]. Okay, next one. Next one. [Moves on to the next problem] There's only one number. There's not two, there's only one number. It is five, so that's fine. But here it's not going to be. So, let's do the same thing. Cross off the tall, short, tall, short. What's left? Good."

In this exchange, Hannah was circulating the classroom during a warm-up activity and attempting to assist students in finding the median. However, her approach involved dictating each step of the procedure rather than providing strategic support that would allow the student to engage independently with the underlying mathematical reasoning. As a result, the instructional interaction limited the opportunity for the student to make sense of the concept or develop problem-solving strategies—key aspects of maintaining a task's cognitive demand (Stein et al., 2009).

In her post-CSPD lesson, Hannah demonstrated increased comfort with granting students greater agency in the problem-solving process. The central task involved determining the landing positions of three birds in the popular game Angry Birds. Students were responsible for identifying the information they needed to solve the problem, which in this case included various forms of quadratic equations. One student chose to use the standard form and apply the quadratic formula—a more complex approach than others available. While Hannah noted that this method might be more challenging, she allowed the student to proceed without discouragement. As the student encountered difficulties, Hannah offered support by posing guiding questions (e.g., "Is there something we can do to get rid of this?" and "If you divide both sides, what happens to that?") rather than providing direct answers. Although she recognized the source of the student's mistake—the student had not applied the order of operations properly to simplify the expression and was left with finding the square root of a negative number, she used strategic questioning to help him uncover and correct the mistake himself.

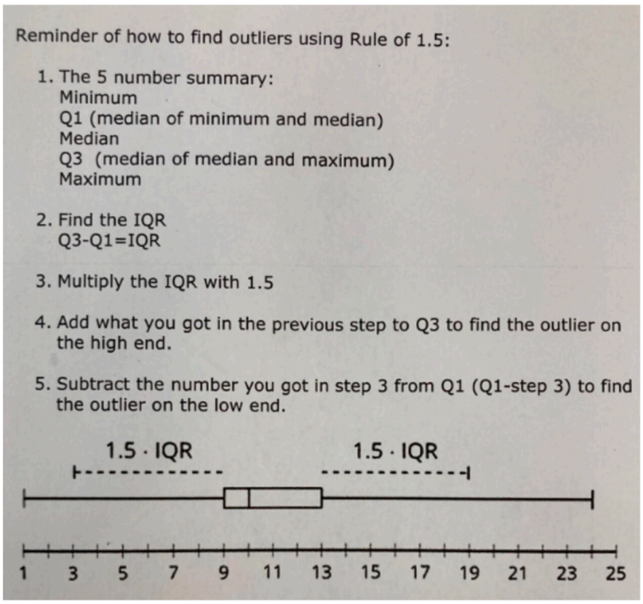


Fig. 6. A step-by-step guide to finding outliers used in a pre-CSPD lesson.

4.3. Student mathematical questioning and reasoning

The code "Student Mathematical Questioning and Reasoning (SMQR)" did not show significant improvements in the mean scores but did show significant improvement in the maximum scores comparison. Fig. 7 shows the results for the seven teachers, with two teachers showing no change and five teachers demonstrating positive increases between the pre- and post-CSPD lessons.

Eva, in her pre-CSPD lesson, relied heavily on direct instruction, offering students limited opportunities to question or reason about mathematical concepts. Student responses were typically brief, focused on providing correct answers, and often formulaic. For instance, in the following excerpt, Eva did all the reasoning about a student's error herself, restricting the student's opportunity to engage in SMQR.

Teacher: Okay, so you have to, how much is this? What is this number? Three or 13.

Student: Three.

Teacher: Three. Okay. Three, and *m* is main dishes, right? Okay. So you have three main dishes. How much is the cost?

Student: Six.

Teacher: Cheap, right? Because you are saying that each one cost seven fifty. And, if you are order or three of them, save more money here,

Student: But…

Teacher: Because if you put three of them, yeah. Main dishes and then the total is six. Then you mean that this cost $2. Well, it's something good. Yeah. For your target. Okay. If you want, you can change that.

Throughout the CSPD, the research team worked with the teacher to develop habits such as asking open-ended questions, allowing students time to grapple with mathematical concepts and models independently, and incorporating strategies like Notice and Wonder to support SMQR. After the CSPD, Eva became more comfortable letting students struggle through concepts, offering appropriate guidance to steer the class toward the lesson's objective while fostering their confidence in SMQR. The dialogue below illustrates this shift over the course of the year. In this example, students worked on a task to find a missing side length of a rectangle, where they were also asked to label each side length as an expression with variables. One student confidently initiated a question about the diagram given by Eva, trying to make sense of it to ensure their understanding of the given task.

Teacher: Put the labels, so you can complete a sketch and label.

Student: So why is it square over there? You said rectangle. Why is it square over there?

Teacher: The square?

Student: Yeah. And I have rectangle.

Teacher: That is good. You put "A" here, right?

Student: Yeah.

Teacher: And then, "minus two" here, right?

Student: That one is [a] rectangle.

Teacher: That is a rectangle, right. It's a small one. That doesn't matter that you have a big one. So, all that part is "A," and this is "A minus two." So, do you have your labels?

As another example, Ellie primarily used direct instruction to convey mathematical content in her pre-CSPD lessons. Her teaching

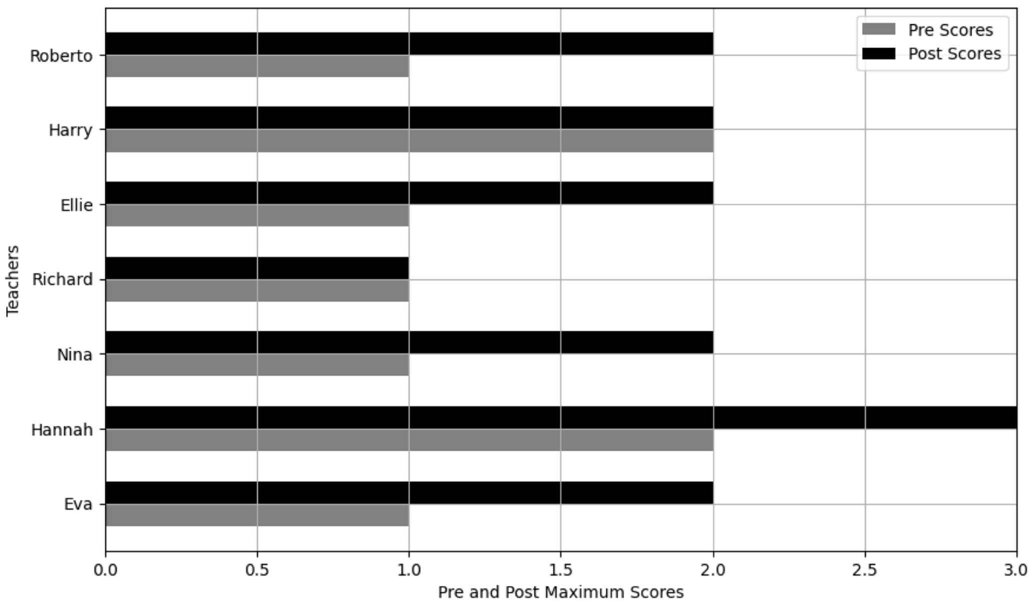


Fig. 7. Maximum scores in pre- and post-CSPD for code "SMQR".

often involved prompting students for rote responses and using their brief answers as a springboard to explain concepts, as illustrated in the following excerpt.

Teacher: Instead of saying three times three times three times three, how many threes do we have?

Student: Four.

Teacher: Four. So I can say three to the fourth. That four is the exponent. So instead of three times, three times three times three, I can say three to the fourth.

In the excerpt, students had limited opportunities to engage with mathematics beyond answering basic questions. In contrast, Ellie's post-CSPD lessons featured tasks that encouraged students to make connections, identify patterns, and generalize their findings to uncover key mathematical ideas. For instance, in a post-CSPD lesson on data representation and analysis, she asked students to share their observations and questions about an image of four dot plots (see Fig. 8).

She assessed students' prior knowledge through their responses and leveraged their natural curiosity to guide them toward the lesson's central concept. By allowing both Spanish and English, she supported student participation and created space for verbal questioning and reasoning, as illustrated in the following exchange.

Teacher: What do we see? What questions do we have for this? [Students noticed the different values on the x-axis, but it was inaudible on the video] The bottom numbers are on the *x*-axis. So this counts by 10. The other counts by ones. How are they similar? You were talking about similar, but not similar.

Student: *No son iguales. El último es más grande que el otro. Casi casi.* [They are not the same. The last one is bigger than the other. Almost.]

Teacher: They're opposite?

Student: *Casi similares.* [Almost similar]

Teacher: …Oh, because of this dot versus this one?

In this exchange, Ellie supports a Spanish-speaking student in articulating observations about the shapes of the distributions on the board. Two of the distributions appear to be near reflections of each other—an idea the student expresses by noting they are almost similar.

4.4. Use of visual aids and supports

The final code showing significant improvement was "Use of Visual Aids and Supports." As seen in Fig. 9, three teachers did not use any visual aids during their pre-CSPD lessons. However, these teachers incorporated visual aids to varying degrees in their post-CSPD lessons.

Ellie demonstrated the most growth between the pre- and post-CSPD lessons, improving her scores from 0 to 3. In the pre-CSPD lesson, she primarily used the whiteboard to demonstrate algorithmic procedures, as shown in Fig. 10.

Throughout the post-CSPD lesson, Ellie incorporated handouts and slides featuring various mathematical representations and visuals to enhance students' understanding of the task. She also actively used the visual display to demonstrate how to find the median

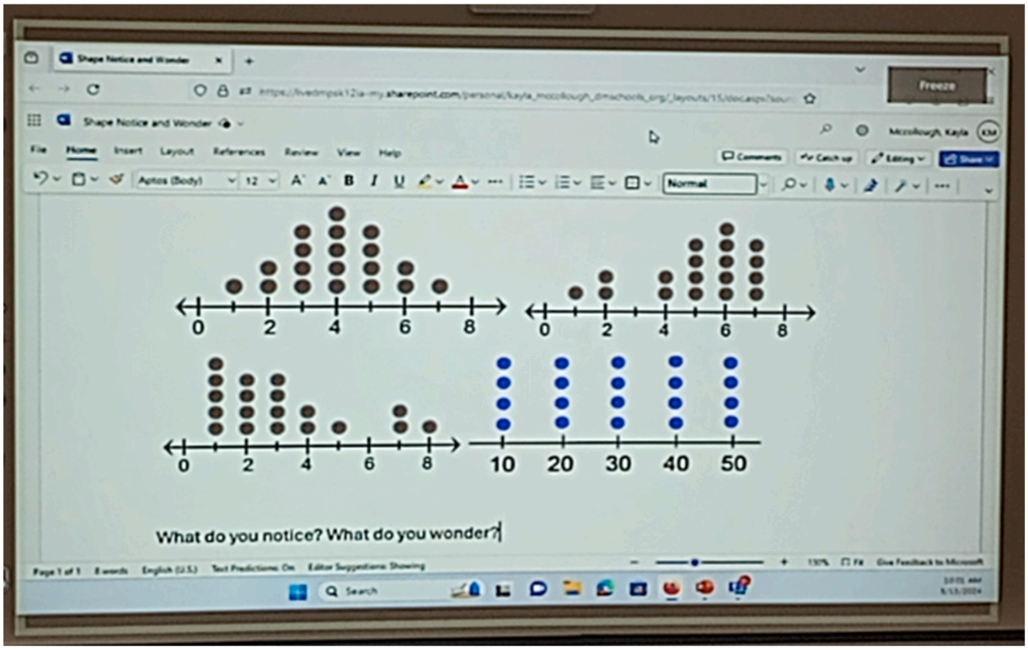


Fig. 8. A notice and wonder activity about distribution shapes.

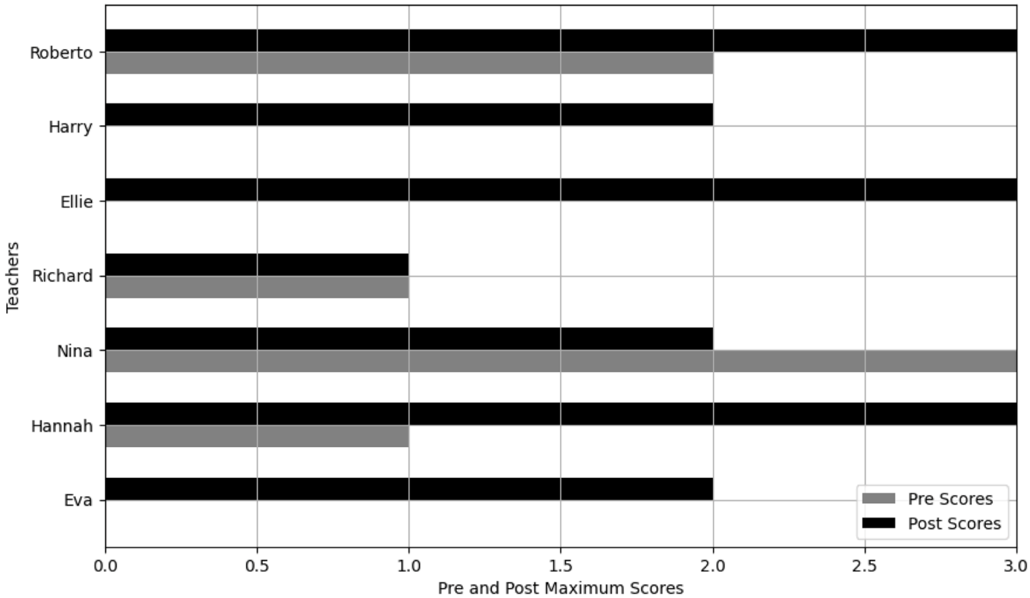


Fig. 9. Maximum scores in pre- and post-CSPD for code "Use of Visual Aids and Support".

of a data set, physically crossing off or covering data points from the outermost to the center. Additionally, Ellie employed visual strategies to reinforce students' understanding of mathematical terminology by highlighting key terms in bold and discussing the meanings of different representations.

Fig. 11 shows the projected handout the teacher used, which featured a contextual image (measuring height) and three different representations: a data set (number list), a number line for creating a dot plot, and a table for constructing a box plot. Ellie invited several students to present their solutions on the board and guided all students in creating both the dot plot and the box plot. Following this, students were asked to discuss how these representations were related. The teacher's notable improvement in the "Use of Visual Aids and Supports" reflects a commitment to dynamic learning and an understanding of the role of visuals in supporting linguistic development.

Eva's case, described in the previous section, also highlights her growing use of visuals. In the pre-CSPD lesson, she relied solely on a workbook and did not incorporate any visual aids. In contrast, during the post-CSPD lesson, she projected a rectangular frame on the screen and drew a rectangle on the board to demonstrate how to represent each side length using a variable. This strategic use of visuals prompted more interaction and discussion with and among students, as reflected in the dialogue presented earlier.

Harry presented another example in which minimal use of meaningful visual representations was observed in the pre-CSPD lesson. His instruction primarily relied on slides that featured definitions and pre-written problems, with limited incorporation of visual supports to enhance students' conceptual understanding. In contrast, during the post-CSPD observation, Harry actively engaged students in constructing a frequency table to represent data they had generated themselves (see Fig. 12). This shift indicated a greater emphasis on supporting students' meaning-making through the use of co-constructed visual representations.

Prior to constructing the frequency distribution, Harry asked each student to write a number between 10 and 100 on the board. These student-generated numbers served as the data set for building the frequency table. Importantly, the table functioned not only as

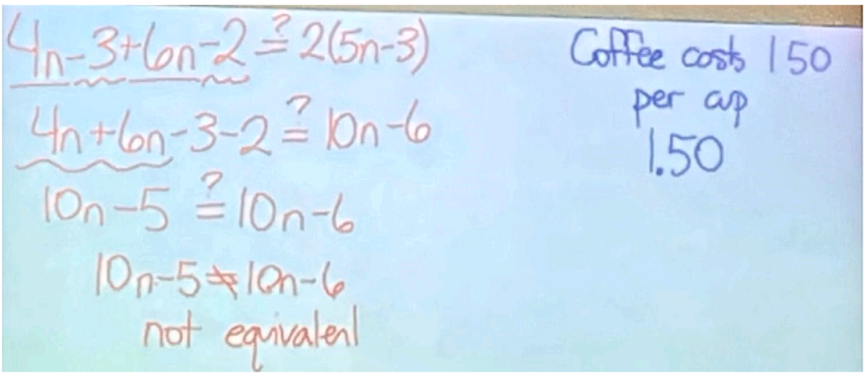


Fig. 10. An example of using visual aids in a pre-CSPD lesson.

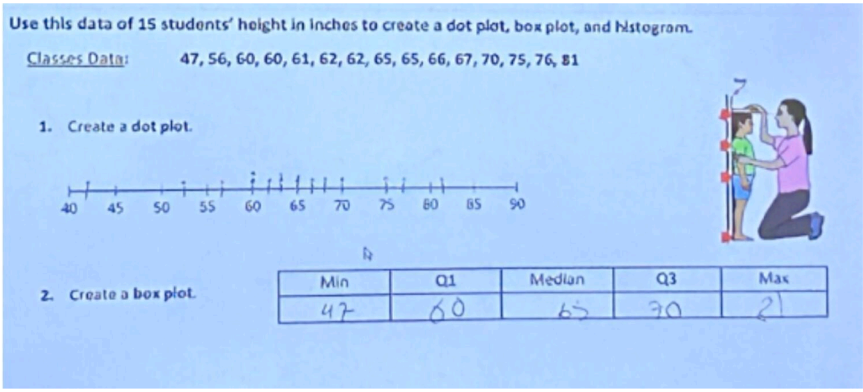


Fig. 11. An example of using visual aids in a post-CSPD lesson.

a visual mathematical model but also as a dynamic instructional tool. Harry actively incorporated gestures and student input in the co-construction of the table, transforming it from a static display into an interactive learning experience. He also used the visual to engage in pre-remediation, helping students recognize and resolve misunderstandings. For instance, he remarked, "Hey. I've heard a number twice, and this is confusing me because we want everything to be in the right place. What's going on with 20? Yeah, if we look here, what's going on with 20? Where does 20 go?" By integrating student contributions into the development of the visual model, Harry supported students in making meaningful connections with the data and in understanding how individual values contribute to overall frequencies in a distribution. This approach reflected a notable shift in his instructional practice, aligning with CSPD goals of promoting conceptual understanding through student-centered and visually supported instruction.

5. Discussions

Our findings, which show improvements in teachers' ability to maintain or increase task cognitive demand, encourage EBs' mathematical and non-mathematical talks in their home language and English, and utilize meaningful visuals, suggest significant growth in teacher knowledge related to supporting EBs in learning mathematics. Research emphasized that the integration of students' familiar language and contexts is essential to engage EBs in cognitively demanding math tasks as well as a critical element of sociocultural theory (Lucas & Villegas, 2013; Moschkovich, 2002). This perspective views mathematical learning as socially and culturally embedded, allowing teachers to use students' existing knowledge as a cultural tool to build bridges to academic math, making content more accessible and meaningful.

From a sociocultural perspective, student talk and visuals are powerful forms of mediation. Increased student talk in either home language or English fosters participation in a community of practice. Language serves as a cultural tool to externalize reasoning for negotiation and refinement (Moschkovich, 2002). Home language talk reduces cognitive load and utilizes a student's full linguistic repertoire for conceptual understanding (Fu et al., 2019). Similarly, visuals act as cultural tools and crucial bridges for learning, enabling EBs to access and grasp mathematical concepts without the full burden of linguistic demands (I & Martinez, 2020). Through these shared tools and social interactions, students build a common understanding within the classroom community.

The participating teachers, though required to use a problem-based curriculum with cognitively demanding tasks, initially provided excessive scaffolding that reduced the cognitive demand by prematurely offering solutions. A notable improvement was observed after their involvement in the CSPD program, as teachers learned to delay immediate solutions, giving students crucial time to explore and make sense of tasks independently. This change is reflected in our coding, with increases in "Students Provide Explanations" and "Student Mathematical Questioning and Reasoning," showing that students were given more opportunities to articulate their thinking and reason using their preferred languages in small groups and whole-class settings. This finding aligns with and elaborates on prior research (e.g., Kraft et al., 2018), which argues that teachers are unlikely to use high-quality curriculum resources to their fullest potential without ongoing professional support like active coaching and side-by-side planning. The observed changes underscore that the curriculum's potential for fostering critical thinking was not fully realized until the teachers received targeted, situated support to navigate the complexities of teaching EBs. Future research should investigate the duration and intensity of support required to ensure that shifts in teacher practice endure after direct coaching concludes.

The improvement in the "Use of Visual Aids or Support" code further illustrates the teachers' growth. Among the 12 coded areas, this code showed the most significant increase in maximum scores. Initially, teachers' use of visuals in pre-CSPD lessons was not observed or often disconnected from the instruction and lacked sustained focus. However, in post-CSPD lessons, teachers demonstrated a more sophisticated understanding of integrating visuals into their lessons in meaningful ways connected to students' life experiences and interests. This result highlights how CSPD effectively increased teachers' ability to use linguistic supports, such as the use of visual aids, which are critical for enhancing EBs' comprehension and engagement (I & Stanford, 2018; I, 2019). While shifts in the other QLDT codes were not statistically significant, we observed upward trends across teachers. We also noted potential benefits of translanguaging practices, especially in bilingual teachers' classrooms. Although not a measured code, the growing encouragement for students to use their full linguistic resources appeared to foster a more supportive environment for sharing mathematical ideas.

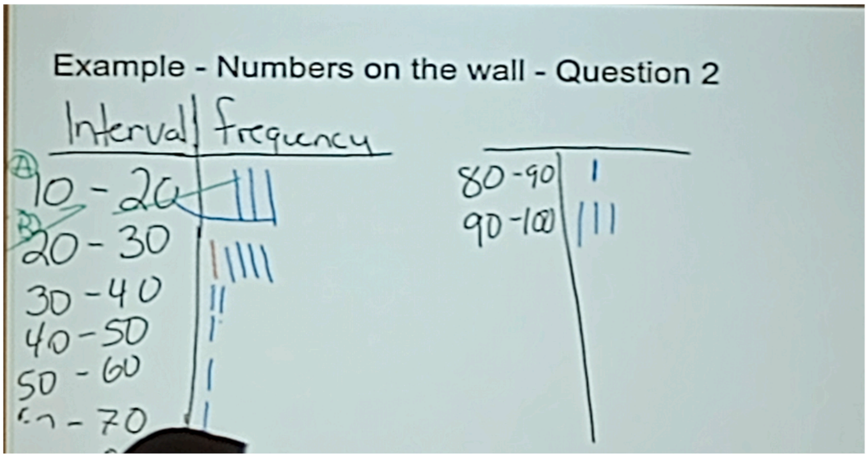


Fig. 12. An example of using student-led visual mathematical model in a post-CSPD lesson.

The teachers in this study came from diverse linguistic backgrounds and ranged from first-year educators to veterans with extensive experience teaching EBs. Despite these differences, all reported limited formal training in mathematics instruction for EBs prior to participating in the CSPD project. The positive impact of CSPD across experience levels highlights its effectiveness in improving instructional practices. These findings suggest that CSPD can be a valuable component of both preservice education and induction programs, helping to better prepare teachers to support EBs. However, sustaining this type of long-term professional development beyond the duration of a research study may pose challenges for school districts. One potential approach to maintaining CSPD is to leverage instructional coaches, district-level curriculum or EB specialists, or experienced CSPD participants as mentors. Partnered with these experts, classroom teachers can continue to engage in CSPD that supports their ongoing learning and practice. Moreover, the varied classroom contexts—from those with a dominant language group to those with multiple languages—demonstrated the adaptability of the CSPD framework, particularly the 5-Act Task structure, in meeting diverse linguistic and instructional needs. Teachers' successful implementation of improved practices across these settings highlights the program's versatility and potential impact.

However, this study has several limitations. While some dimensions of instructional quality showed noticeable improvement, many QLDT codes did not exhibit significant change. This may be due to the fact that most of these dimensions—except for life connections—were not explicitly addressed in the CSPD, although they may have been implicitly supported. In some cases, teachers may have already been incorporating these practices into their instruction. The small sample size and the extended duration of the CSPD also limit the generalizability of the findings. Future research should involve a larger and more diverse group of teachers and expand the CSPD model to explicitly target a broader range of QLDT dimensions. It should also integrate additional linguistic components, such as translanguaging. Although teachers in this study used meaningful translanguaging strategies—for example, allowing students to draw on their full linguistic repertoires to make sense of mathematics—these practices were not explicitly captured by either the MQI or QLDT coding schemes. Future studies should examine how translanguaging is taken up through CSPD and how it supports emergent bilinguals' conceptual understanding and classroom participation. Additionally, because this study was conducted exclusively in classrooms composed entirely of EBs, its findings may not reflect the realities of more common mixed classrooms that include both EBs and non-EBs. Future research should explore how CSPD functions across different classroom compositions and investigate its impact on EBs' engagement, participation, and development of mathematical identities, offering deeper insights into the long-term effects of CSPD on student learning.

Finally, it is important to note that the pedagogical practices promoted in this study—such as maintaining cognitive demand, encouraging student explanations, and employing visual supports—are not only effective for EBs but also enhance learning opportunities for all students. Research has consistently shown that maintaining high cognitive demand in mathematics tasks supports deeper conceptual understanding (Parrish & Bryd, 2022). Encouraging students to articulate their reasoning promotes mathematical discourse and metacognition, which are critical for developing robust mathematical understanding across diverse student populations (Boaler & Staples, 2008). Similarly, the use of visual representations and other multimodal supports helps all learners access complex ideas by making abstract concepts more concrete (Moschkovich, 2013). Therefore, while the CSPD framework in this study focused on improving instruction for EBs, its core pedagogical principles are broadly applicable and contribute to more inclusive and effective mathematics classrooms for all students.

Funding Detail

This work was supported by the National Science Foundation under Grant A# 1941668.

Declaration of Competing Interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Jiyeong Yi reports financial support was provided by National Science Foundation. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share data.

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